

# Rigidity dependence of cosmic ray escape length in the Galaxy obtained from a comparison of proton and iron spectra in the range 3-3000 GV

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**Abstract.** The simple leaky-box model of propagation of cosmic rays in the Galaxy is quite suitable for handling data on cosmic ray nuclei energy spectra and composition at  $E \gg 1$  GeV [1,2]. In the leaky-box model a full information about cosmic ray propagation in Galaxy is compressed to the single parameter - escape length,  $X_e$ , characterizing mean grams of a matter passed by cosmic rays from sources to the Earth. In this paper we analyze the world data on proton and iron cosmic ray spectra collected in the past (HEAO, CRN et al.) and in series of recent electronic experiments (ATIC, CREAM, AMS, BESS, Tracer et al.) and obtain the rigidity dependence of escape length,  $X_e(R) \sim R^{-0.47 \pm 0.03}$ , from the measured rigidity dependence of the protons/iron ratio. It quite agrees with the one estimated in standard manner from the secondary/primary nuclei ratio. But at  $R > 300$  GV the behavior of  $X_e(R)$  distinctly changes, that can (variant of explanation) point out to the change of proton/iron ratio in cosmic ray sources.

**Keywords:** cosmic rays, propagation, escape length

## I. INTRODUCTION

In spite of apparent absence of physical background for the leaky-box model, where the transport of energetic particles is described by introducing the mean escape time of cosmic rays from the Galaxy, and the cosmic-ray density, the source density, the gas density do not depend on coordinates, it may be applied to the study of diffusion and nuclear spallation of stable nuclei attended by the production of secondary relativistic nuclei in the interstellar gas [1] [2]. This can be explained by the concentration of cosmic rays sources and the interstellar gas in a relatively thin galactic disk immersed in the flat but thick cosmic-ray halo [1]. The spatial distribution in the low-density halo is the same for different stable nuclei because of the negligible nuclear spallation. The calculated intensities of stable nuclei for an observer at the galactic disk look as corresponding leaky-box expressions even for nuclei with large cross sections and all propagation is described by some escape length of

cosmic rays from the Galaxy  $X_e$  (measured in g/cm<sup>2</sup>) that is a function of particle rigidity  $R$ . This important parameter means grams of a matter passed by cosmic rays from sources to the Earth in average. In [2] there was proposed a way to estimate the analogous parameter in the diffusion model - effective values of  $X_{ef}$ . It was shown that leaky-box is a good approximation to the widely known and used for the interpretation of various cosmic-ray data basic GALPROP (Galactic Propagation) model [3] with 1% accuracy for all nuclei. In [2] values of  $X_{ef}$  were found for the three set of GALPROP parameters, corresponding to the three models of the propagations. 1) Plain diffusion model (PD)

$$X_{ef} = 19\beta^3 \text{ g/cm}^2 \text{ } R < 3 \text{ GV},$$

$$X_{ef} = 19\beta^3 (R/3 \text{ GV})^{-0.6} \text{ g/cm}^2, R > 3 \text{ GV}; \quad (1)$$

2) Diffusion with acceleration (DR)

$$X_{ef} = 7.2(R/3 \text{ GV})^{-0.34} \text{ g/cm}^2, R > 30 \text{ GV}; \quad (2)$$

3) Diffusion with damping (DRD)

$$X_{ef} = 13(R/3 \text{ GV})^{-0.5} \text{ g/cm}^2, R > 10 \text{ GV} \quad (3)$$

For the comparison we present also the widely used approximation [4]:

$$X_{ef} = 11.8\beta(R/4.9 \text{ GV})^{-0.54} \text{ g/cm}^2, R > 4.9 \text{ GV} \quad (4)$$

The leaky-box approach due to its simplicity until very recent was used by many authors.

Intensity  $I_A$  of CR nuclei with mass number  $A$  near the Earth is connected with their spectrum in a source  $Q_A$  (if neglecting the energy losses at small energies and the contribution from the fragments produced from heavier nuclei) by the simple equation:

$$I_A = \frac{Q_A(R)}{4\pi\rho} \times \frac{1}{\frac{1}{X_{ef}} + \frac{1}{X_{in}^A}}, \quad (5)$$

TABLE I: The fit of the ratio of  $I_H/I_{Fe}$  – fit by the polynomial function of the fifth order. Averaged points of the ratio  $I_H/I_{Fe}$  – points are calculated in 14 R-bins with the corresponding errors

R, GV	3.16	5	8	12.6	20	31.6	50	89	178	355	708	1412	2818
$I_p/I_{Fe}$ – fit	2907	2313	1875	1564	1347	1197	1095	1012	952	914	889	877	894
Errors	249	212	105	47	43	101	41	56	65	84	143	166	150
$I_p/I_{Fe}$ – points	2432	2281	1748	1676	1235	1203	1067	991	842	1053	955	976	1274
Errors	249	212	105	50	40	101	40	54	57	97	154	185	447

where  $\rho$  – gas density,  $X_{in}^A$  – interaction length for the nucleus with mass number  $A$ . (calculated from GALPROP). For protons  $X_{in}^H \sim 110$  g/cm<sup>2</sup> at 1 GeV and it decreases to 70 g/cm<sup>2</sup> at 10000 GV, that is  $X_{ef} \ll X_{in}^A$ , from whence the well known equation arises:

$$I_H \sim Q_H * X_{ef}, \quad \gamma_{obs} = \gamma_{sour} + \alpha,$$

where  $\alpha$  is the slope of  $X_{ef}$  dependence,  $\gamma_{obs}$ ,  $\gamma_{sour}$  – the slopes of observed and source spectra correspondingly in the case of power-like laws.

But while interaction length for the iron nuclei is much smaller  $X_{in}^{Fe} \sim 2.7$  g/cm<sup>2</sup>, there is the rigidity region  $R_{min} \div R_{max}$  for Fe nuclei where

$$I_{Fe} \sim Q_{Fe} * X_{in}^{Fe}, \quad \gamma_{obs} \sim \gamma_{sour},$$

reaching the asymptotic value (proton and Fe spectra are parallel),  $\gamma_{obs} = \gamma_{sour} + \alpha$  when  $X_{ef}$  at  $R_{max}$  becomes much smaller than  $X_{in}^{Fe}$ .

We see that one can try to estimate the rigidity dependence of  $X_{ef}$  from the ratio of any two spectra in the interval  $R_{min} \div R_{max}$ , but with essential reservations:

- to bear in mind that the intrinsic property of the leaky-box model is the independence of  $X_e$  on cross section and as a result on the type of nucleus;
- to assume that all types of nuclei are produced in the same type of sources and the chemical composition of accelerated particles in the sources does not depend on rigidity in the investigated interval -

$$Q_{Fe}(R)/Q_H(R) = \text{const};$$

- to use nuclei with significantly different interaction lengths, besides, the contribution of fragments among them should be negligible;
- to find way to estimate  $Q_{Fe}(R)/Q_H(R)$ .

Proton and iron spectra are most suitable ones for this task.

The main question is: if available in our days experimental data are enough for this task?

## II. EXPERIMENTAL DATA

As we are going to analyze spectra at fixed rigidity, we need the spectral data measured at remote intervals of energy per particle. Interval pointed out in title, 3-3000 GV, corresponds to the energy per particle interval 2.3 GeV - 3 TeV for protons and 44 GeV - 82 TeV for iron nuclei. So we are doomed to use data from different experiments. Moreover only with appearance of new data on iron nuclei from the experiment Tracer [19], measured in uniquely wide energy range ( $E_{part}=30$

GeV-80 TeV), this task probably can be solved. Data obtained in ATIC2 experiment [11] are very important also, because they fill up the gap between data obtained by magnetic spectrometers and ones obtained with calorimeters and emulsions.

We include in the consideration experimental points, which satisfy the conditions:

- a) energy interval lies between 3 and 10000 GV,
- b) errors don't exceed 30%,
- c) calibration on accelerators is done (that is why Sokol experiment was excluded).

Fig. 1 and Fig. 2 represent proton and Fe spectra measured in different experiments. The points satisfying above enumerated conditions are denoted by filled symbols.

Proton data are got from the following experiments: MASS91 [5], BESS-98 [6], [7]BESS02, AMS [8], [9], [10], ATIC02 [11], CREAM [12], JACEE [13], [14], RUNJOB [15]. Fe nuclei are from: HEAO-3 [16], CRN [17], TRACER [18], [19], [11], Sanriku [20]. Data were reduced to fixed modulation potential  $\Phi = 800$  MV by calculation similar to [10].

Fortunately points satisfying above mentioned conditions are little scattered. For further work and convenience one needs to approximated these dependencies by any way. In the Fig. 1 and 2 we denote by thick line the best fit of corresponding scatter points, that is a polynomial function of the fifth order. In the Table 1 this fit is denoted as " $I_H/I_{Fe}$  – fit" with errors in the upper two rows.

Moreover, we calculated averaged points in the every from 14 R-bins getting errors which include statistical significance and a divergence of experimental points fallen in the fixed bin (thin lines with errors in Fig. 1,2.). In the Table 1 averaged points are denoted as " $I_H/I_{Fe}$  – points" and they are presented in the lower two rows.

## III. RESULTS AND DISCUSSION

The leaky-box Ed. (5) written for the proton and Fe spectra allows us to express the value of effective escape length  $X_{ef}$  by means of the ratio of proton/iron measured spectra  $I_H/I_{Fe} = I_H/I_{Fe}(R)$  (from Table 1), the source chemical composition  $Q_{H/Fe} = Q_H/Q_{Fe}$  (which does not depend on rigidity and should be determined any how) and known values of  $X_{in}^{Fe}$ ,  $X_{in}^H$ .

$$X_{ef} = \frac{I_H/I_{Fe}(R)/Q_{H/Fe} - 1}{\frac{1}{X_{in}^{Fe}} + \frac{I_H/I_{Fe}(R)/Q_{H/Fe}}{X_{in}^H}}. \quad (6)$$

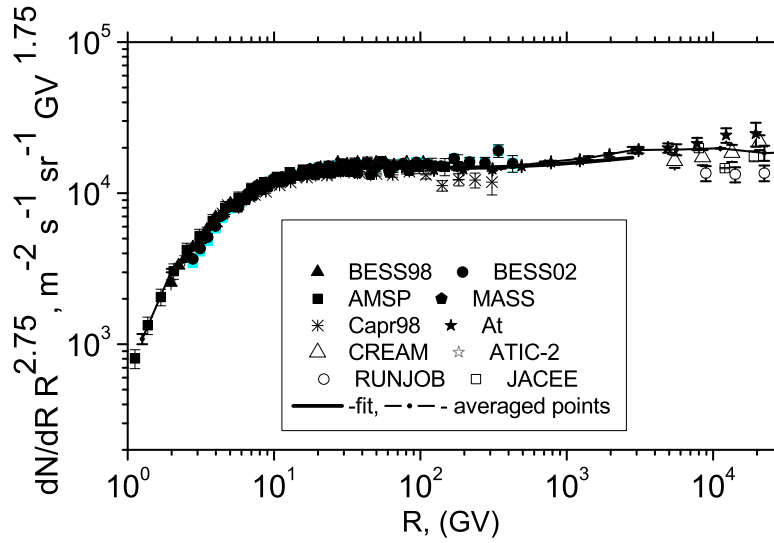


Fig. 1: Proton spectra measured in different experiments (for ref. see text), reduced to the modulation potential  $\Phi = 800$  MV; open symbols - points with more than 30%; thick line - the 5th order polynomial fit, thin lines - averaged points

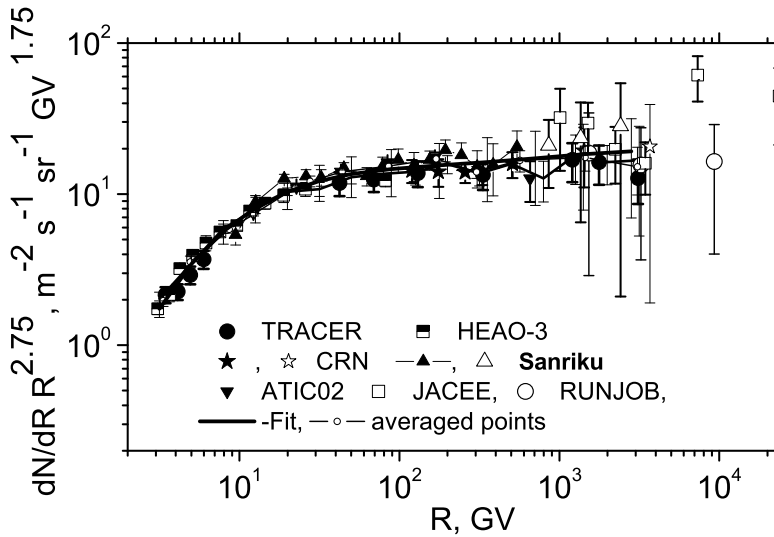


Fig. 2: Spectra of iron nuclei; open symbols - points with more than 30%; thick line - the 5th order polynomial fit, thin lines - averaged points

The Eq. (6) will work obviously only in the range of rigidities  $R_{min} - R_{max}$  where  $X_{ef}$  is not much smaller than  $X_{in}^{Fe}$ : at  $X_{ef} \rightarrow 0$   $I_{H/Fe}(R) \rightarrow Q_{H/Fe}$ . To demonstrate expected application range of Eq. (6) we choose the critical value  $R_{max}$  corresponding to  $X_{ef} = 0.1X_{in}^{Fe} \sim 0.27\text{g/cm}^2$ . It means that the region  $X_{ef} < 0.27\text{g/cm}^2$  and  $R > R_{max}$  is the asymptotic region where spectra become parallel and not sensitive to the  $X_{ef}$ . For three models embedded in GALPROP (see Introduction) the corresponding values of  $R_{max}$  look like below:

- 1)  $R_{max} = 3.6$  TV for PD model( $\alpha = -0.6$ ),
- 2)  $R_{max} = 6.9$  TV for DRD model( $\alpha = -0.5$ ),

- 3)  $R_{max} = 46.8$  TV for DR model( $\alpha = -0.34$ ),
- 4)  $R_{max} = 69000$  TV for RD model( $\alpha = -0.2$ ) for the very flat energy dependence  $X_{ef} = 10R^{-0.2}$  considered in [21] being chosen for the explanation of the "knee" in PCR by the change of propagation mechanism.

So  $R_{max}$  comparable with the experimental maximal  $R = 2.8$  TV (see Table 1) may be only for very steep  $X_{ef}$  dependencies, as in the cases of PD or DRD models. In this case we determine the value of  $Q_{H/Fe}$  from the asymptotic ratio of  $I_{H/Fe}(R)$  taking into account minor corrections. We estimate  $Q_{H/Fe} = 800 - 1000$ .

The variant of calculation by the Eq. (6) for the experimental values of  $I_{H/Fe}(R)$  from the Table 1 and

$Q_{H/Fe}=800$  give the next approximation:  $X_{ef}(R) \sim 4.6 * (R/5 \text{ GV})^{-0.65}$ , that is much lower than the expected dependence  $X_{ef}$  estimated by the  $B/C$  ratio (1)-(4). This means that our assumption of steep  $X_{ef}$  decrease with energy  $R > 3 \text{ TV}$  (as  $R^{-0.5 \div -0.6}$ ) is not fully correct.

For the analysis of the set of flat dependencies  $X_{ef}(R)$  where  $R_{max} \gg 3 \text{ TV}$  and there is no possibility to estimate the asymptotic values  $I_{H/Fe}(R)$  it is proposed to "normalize"  $X_{ef}$  at  $R = 50 \text{ GV}$  by the one obtained from  $B/C$  measurements. The values of  $X_{ef}^{B/C}$  obtained from  $B/C$  - ratio occur between  $2.8 \text{ g/cm}^2$  (DR model - (2)) and  $3.45 \text{ g/cm}^2$  (PD model (1)). The point  $R = 50 \text{ GV}$  was chosen because the large body of data on  $B/C$  are in a good coincidence in this point [22], from the other side at the  $R = 50 \text{ GV}$  the contribution of reacceleration processes surely should be small.

Then  $Q_{H/Fe}$  value could be determined using  $X_{ef}^{B/C}(50\text{GV}) \approx 2.8 - 3.45 \text{ g/cm}^2$ , and  $I_{H/Fe}(R)(50\text{GV}) = 1067 \pm 40$  (from Tabl. 1). By Eq. (6) we get the corresponding interval for  $Q_{H/Fe} = 490 - 535$ . Substituting obtained limit values of  $Q_{H/Fe}$  and measured rigidity dependence of proton/iron spectra from Tabl. 1 in Eq. (6) we calculate two variants of rigidity dependencies of  $X_{ef}(R)$ , presented in Fig. 3. The upper one (with  $Q_{H/Fe}=490$ ) could be approximated in the interval  $R = 3 - 300 \text{ GV}$  as  $X_{ef}(R) \sim 3.46 * (R/5 \text{ GV})^{-0.47 \pm 0.03}$ . The lower one (with  $Q_{H/Fe}=535$ ) could be approximated as  $X_{ef}(R) \sim 2.7 * (R/5 \text{ GV})^{-0.50 \pm 0.03}$ .

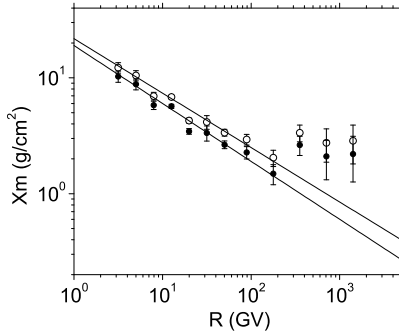


Fig. 3:  $X_{ef}(R)$  dependencies calculated from the measured proton/iron spectra (lowest two rows in Tab. 1) for the  $Q_{H/Fe}=535$  (black circles),  $Q_{H/Fe}=490$  (open circles). Thin lines - the approximations by power law.

Up to the  $R \sim 300 \text{ GV}$  the obtained from protons/iron ratio rigidity dependence of escape length  $X_{ef}(R)$  can be approximated by the power-like law:  $X_{ef}(R) \sim R^{-0.47 \pm 0.03}$ , that is in a good agreement with the one estimated in standard manner from secondary/primary nuclei ratio (1)-(4). But at  $R > 300 \text{ GV}$  the behavior of  $X_{ef}(R)$  changes significantly. It is caused by the "improper" behavior of the slopes of proton and iron spectra at  $R > 100 \text{ GV}$ ,  $\gamma_{Fe} = 2.71 \pm 0.03$  [19] and  $\gamma_H = 2.63 \pm 0.03$  [11]. They are in a striking contradiction with the

expected values: the Fe spectrum should be flatter than proton spectrum (see Eq.(5)).

Here it is worth noting that in the region  $300 \text{ GV}-3 \text{ TV}$  (Fig.1, 2) the principal contribution for protons comes from ATIC2 data [11] and for iron nuclei it comes from the Tracer data [19].

So if we believe that both these important experiments are reliable, we should inevitably conclude that in the range  $R > 300 \text{ GV}$  there is a change of proton/iron ratio in cosmic ray sources. Some authors of ATIC have come to this conclusion already [23].

In summary we would like to stress that the region  $300 \text{ GV} - 10 \text{ TV}$  continues to be of importance, and the result that spectrum of iron nuclei is flatter than proton spectrum needs to be confirmed in other experiments.

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